

NSF-ITP-95-50
hep-th/9506048

Moduli Space of CHL Strings

Shyamoli Chaudhuri* and Joseph Polchinski†

Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106-4030

Abstract

We discuss an orbifold of the toroidally compactified heterotic string which gives a global reduction of the dimension of the moduli space while preserving the supersymmetry. This construction yields the moduli space of the first of a series of reduced rank theories with maximal supersymmetry discovered recently by Chaudhuri, Hockney, and Lykken. Such moduli spaces contain non-simply-laced enhanced symmetry points in any spacetime dimension $D < 10$. Precisely in $D=4$ the set of allowed gauge groups is invariant under electric-magnetic duality, providing further evidence for S-duality of the $D=4$ heterotic string.

*sc@itp.ucsb.edu

†joep@itp.ucsb.edu

1 Introduction

Toroidal compactification of the heterotic string [1] preserves the full spacetime supersymmetry, giving the algebras $N=4$ in $D=4$, $N=2$ (nonchiral) in $D=6$, and $N=1$ in $D=8$. These maximally supersymmetric compactifications have played a major role in recent discussions of strongly coupled string theory.¹

Banks and Dixon[5] showed that maximal supersymmetry in the heterotic string requires that the right-moving (supersymmetric) target space be a $(10-D)$ -dimensional torus. For some time the only known example was that of ref. [1] in which the left-moving degrees of freedom were also toroidal. Recently, Chaudhuri, Hockney, and Lykken (CHL) have used fermionization to construct many new exact conformal field theory solutions² having maximal supersymmetry in any spacetime dimension $D<10$ [6]. These solutions are characterized by a reduction of the rank of the left-moving gauge group relative to the toroidally compactified ten dimensional heterotic string. CHL have found maximally supersymmetric $N=4$ theories in $D=4$ spacetime dimensions with gauge groups of rank

$$(r_L, r_R) = (26 - D - k, 10 - D), \quad k = 8, 12, 14, 16, 18, 20, 22. \quad (1)$$

Here k is the reduction of the rank. The first two members of this series can be followed up to (even) spacetime dimension $D=8$ and $D=6$, respectively. Since the moduli in these maximally supersymmetric theories appear in vector multiplets, the number of inequivalent marginal deformations at any point in the moduli space is fixed to be $r_L r_R$. It was shown in [6],[7] that such moduli spaces contain points of both non-simply-laced and simply-laced enhanced symmetry, as well as higher level realizations of the gauge symmetry.³

¹For reviews and recent discussions see refs. [2], [3], [4].

²We will use the term “solutions” since it not clear at present whether all solutions to the heterotic string consistency conditions can be interpreted as backgrounds or compactifications of the ten-dimensional string, though this is widely assumed.

³We should also note that analogous solutions for the Type II string, four dimensional theories with extended spacetime supersymmetry including $N=4_{R+0_L}$, 4_{R+1_L} , 4_{R+2_L} , and 2_{R+2_L} , and with reduced dimension moduli spaces, were discovered some time ago by Ferrara and Kounnas using the free fermionic construction [8]. The Type II solutions differ from those of [6] in using left-right symmetric Majorana fermions, which are excluded in the heterotic case by maximal supersymmetry [5]. The heterotic solutions require chiral Majorana fermions. A target space interpretation of these Type II conformal field theory backgrounds, and the identification of their duals, remains to be explored. We would like to thank Costas Kounnas for informing us of this work.

The purpose of this paper is to clarify the nature of these theories. While fermionization has proven to be a powerful technique for discovering qualitatively new classes of string compactifications it can give only isolated pieces of their moduli space, generally at points of higher symmetry. We will describe an orbifold construction which yields the moduli space of the first theory in this series, recovering, in particular, all of the $k = 8$ solutions given by the fermionic construction. We also discuss the implications for strong-weak coupling duality in string theory. For the conjectured self-duality of the heterotic string in $D = 4$ we find new evidence: gauge groups always appear with their electric-magnetic duals, for example $Sp(20) \times SO(9)$ and $Sp(8) \times SO(21)$ in the same moduli space. However, the CHL theories present a puzzle for the conjectured string-string duality in $D = 6$.

2 Construction

A clue to the nature of these theories comes from considering the decompactification limit. Moduli $j\tilde{j}$ can always be constructed from the Cartan subalgebra of the gauge group. At large values of these moduli the spectrum becomes continuous, and as long as the left-moving rank is at least $10 - D$ one can reach in this way a theory with 10 large translationally invariant dimensions, which must be a toroidal compactification of either the $E_8 \times E_8$ or $SO(32)$ heterotic string. This seems a puzzle at first, since the construction of ref. [1] appears to be general, but there is at least one additional possibility. Consider the effect of transporting a string around a toroidal dimension. The string must return to its original state up to a symmetry g . For g a general gauge transformation (inner automorphism of the gauge group), this is equivalent to a Wilson line background and is therefore included in the classification of ref. [1].

However, the $E_8 \times E_8$ theory also has an outer isomorphism which interchanges the two E_8 's.⁴ Modding out by the outer isomorphism alone does not lead to a new theory, because the gauge bosons of the other E_8 are recovered in the twisted sector. The only additional possibility in $D = 10$ is to combine this action with a non-trivial twist on the world-sheet fermions, but this breaks the spacetime supersymmetry to $N=0$ [9]. Since our interest is in Z_2 actions that leave the supersymmetry unbroken, twists on the world-sheet fermions are excluded. Compactifying on a torus to any spacetime

⁴The outer isomorphism has been considered previously in refs. [9], and very recently in [10], but always in conjunction with right-moving twists which reduce the spacetime supersymmetry.

dimension $D < 10$ opens up a new option: an accompanying translation in the torus. Begin with an ordinary toroidal compactification to D spacetime dimensions and twist by RT , where R is the outer isomorphism that interchanges the two E_8 lattices and T is a translation in the spacetime torus. This has just the necessary action: it eliminates one linear combination of the two E_8 's, leaving the diagonal E_8 at level 2.

By a similarity transformation T and R can be taken to commute. Without loss of generality we assume that $(RT)^2 = T^2$ is a symmetry of the original lattice, else we could twist by T^2 to obtain a different lattice in the same moduli space. Also we can assume that T is not a symmetry of the lattice; if it were then RT would be equivalent to R and so would actually act trivially.

Denote a general momentum state by $|p_1, p_2, p_3\rangle$. Here

$$p_1^I = \frac{1}{\sqrt{2}}(p^I - p'^I), \quad p_2^I = \frac{1}{\sqrt{2}}(p^I + p'^I), \quad I = 1, \dots, 8 \quad (2)$$

are the linear combinations of momenta in the two E_8 lattices which are respectively reflected and left invariant by R , while

$$(p_3^m; \tilde{p}_3^m), \quad m = D, \dots, 9 \quad (3)$$

are the momenta of the torus. Taking T to be a translation by $2\pi(0, v_2^I, v_3^m; \tilde{v}_3^m)$, RT acts as

$$RT|p_1, p_2, p_3\rangle = e^{i2\pi v \cdot p} | -p_1, -p_2, -p_3\rangle \quad (4)$$

where the inner product has signature $(26 - D, 10 - D)$.

We start with the $E_8 \times E_8$ theory, with $10 - D$ dimensions compactified on a given torus without background gauge fields. The momentum lattice of such a theory is of the form

$$\Gamma = \Gamma^8 \oplus \Gamma^8 \oplus \Gamma', \quad (5)$$

where Γ' , is even, self-dual, and of Lorentzian signature $(10 - D, 10 - D)$. In the basis above Γ takes the form

$$\left| \frac{1}{\sqrt{2}}(p^I - p'^I), \frac{1}{\sqrt{2}}(p^I + p'^I), p_3 \right\rangle, \quad (6)$$

where $p^I, p'^I \in \Gamma^8$ and $p_3 \in \Gamma'$. Specifically,

$$\Gamma^8 : \quad p^I = \frac{1}{2}m^I, \quad m^I \in \mathbf{Z}, \quad m^I - m^J \in 2\mathbf{Z}, \quad \sum_I m^I \in 4\mathbf{Z}. \quad (7)$$

General points in the moduli space can then be reached by boosts of the momentum lattice as in ref. [1]. In order to allow a twist by R the boost Λ must commute with R , leaving p_1 invariant.⁵ This subgroup is $SO(18-D, 10-D)$, so the moduli space of inequivalent vacua is locally of the form

$$\frac{SO(18-D, 10-D)}{SO(18-D) \times SO(10-D)} \quad (8)$$

as required by considerations of low energy supergravity.

The twist by RT produces an asymmetric orbifold [11]. We review the relevant results from that paper. The lattice I is defined to consist of those momenta invariant under R . Here, this implies

$$I : \quad p_1 = 0, \quad p_2 \in \sqrt{2}\Gamma^8, \quad p_3 \in \Gamma'. \quad (9)$$

The dual lattice, in the subspace invariant under R , is

$$I^* : \quad p_2 \in \frac{1}{\sqrt{2}}\Gamma^8, \quad p_3 \in \Gamma'. \quad (10)$$

The number of twisted sectors is D where

$$D^2 = \det'(1-R)|I/I^*| = 2^{8-8} = 1. \quad (11)$$

The momenta in the twisted sector are

$$p \in I^* + v \quad (12)$$

and the twisted sector spectrum consists of the level-matched states. The untwisted sector simply retains the RT -invariant states. The left-moving zero point energies are the usual $-\frac{24}{24} = -1$ in the untwisted sector and $-\frac{16}{24} + \frac{8}{48} = -\frac{1}{2}$ in the twisted sector.

2.1 Compactification to $D = 8$

Consider first the case $D = 8$. Let $\Gamma' = \Gamma^{SU(2)} \oplus \Gamma^{SU(2)}$, two copies of the free boson at the $SU(2)$ radius:

$$\Gamma^{SU(2)} : \quad p_3 = \frac{1}{\sqrt{2}}(n; \tilde{n}), \quad n, \tilde{n} \in \mathbf{Z}, \quad n + \tilde{n} \in 2\mathbf{Z}. \quad (13)$$

⁵More generally there is the possibility that $\Lambda R \Lambda^{-1}$ be a nontrivial discrete symmetry (duality) of Γ , leading to a disconnected moduli space. We will not consider this here.

We take T to be a translation by half the spacetime periodicity in the $m=9$ direction,

$$v_2 = 0, \quad v_3 = \left(0, \frac{1}{2\sqrt{2}}; 0, -\frac{1}{2\sqrt{2}}\right). \quad (14)$$

Let us determine the gauge symmetry. The untwisted sector contains ten neutral left-moving gauge bosons⁶

$$\frac{1}{\sqrt{2}}(\alpha_{-1}^I + \alpha_{-1}'^I)|0, 0, 0\rangle, \quad \alpha_{-1}^m|0, 0, 0\rangle, \quad m = 8, 9, \quad (15)$$

the eight antisymmetric neutral combinations being removed by the RT projection. Let r denote any root of E_8 . The charged gauge bosons in the $E_8 \times E_8$ theory which are invariant under RT are the symmetric combinations

$$\frac{1}{\sqrt{2}}\left(\left|\frac{1}{\sqrt{2}}r, \frac{1}{\sqrt{2}}r, 0\right\rangle + \left|-\frac{1}{\sqrt{2}}r, \frac{1}{\sqrt{2}}r, 0\right\rangle\right) \quad (16)$$

and the $SU(2)$ gauge bosons

$$p_1 = p_2 = 0, \quad p_3 = \left(\pm\sqrt{2}, 0; 0, 0\right). \quad (17)$$

In the twisted sector there are no massless states because the right-moving component of the momentum (12) will not vanish. The states (16) form the E_8 root lattice, but the momentum p_2 to which the neutral gauge bosons (15) couple is scaled down by a factor of $\sqrt{2}$ so the current algebra realization of E_8 is at level 2. The gauge bosons (17) are the remnant of one of the original $SU(2)$'s, the other being removed by the RT projection, so that the gauge symmetry originates in the left-moving current algebra $(E_8)_2 \times SU(2)_1 \times U(1)$.

General points in the moduli space are reached by $SO(10, 2)$ boosts. Generically the left-moving symmetry is broken to $U(1)^{10}$, but there are also points of enhanced gauge symmetry. Consider for example the vectors

$$\begin{aligned} e_{(1)} : \quad & p_1 = p_2 = 0, \quad p_3 = \sqrt{2}(1, 0; 0, 0), \\ e_{(2)} : \quad & p_1 = p_2 = 0, \quad p_3 = \frac{1}{\sqrt{2}}(0, 3; 2, 1). \end{aligned} \quad (18)$$

These survive the RT projection in the untwisted sector and satisfy

$$e_{(i)} \cdot e_{(j)} = 2\delta_{ij}. \quad (19)$$

⁶The right moving gauge symmetry is always $U(1)^{10-D}$ in a maximally supersymmetric compactification of the heterotic string [5].

Making an $SO(2, 2)$ boost to a frame in which $\frac{1}{\sqrt{2}}e_{(i)}$ form a left-moving orthonormal basis, these become $SU(2) \times SU(2) = SO(4)$ weights. The vectors

$$\pm \frac{1}{2}e_{(1)} \pm' \frac{1}{2}e_{(2)}, \quad (20)$$

where all four independent choices of sign are included, appear in the twisted sector (12). These momenta have length-squared 1 and so with the twisted zero-point energy give rise to massless states, combining with $\pm e_{(i)}$ to form the root lattice of $SO(5)$. The left-moving current algebra is $(E_8)_2 \times SO(5)_1$.

Notice in this example that short roots of a level 1 non-simply-laced algebra and long roots of a level 2 algebra both have $p_2 \cdot p_2 + p_3 \cdot p_3 = 1$, and that they can arise in the Hilbert space in two ways: as untwisted states with momentum $p_1 \cdot p_1 = 1$ or as twisted states. Note also that in this moduli space non-simply-laced algebras can only appear with root lengths in the ratio $1 : \sqrt{2}$, and only as level 1 realizations, and that simply-laced algebras can appear only at levels 1 or 2.

To find further enhanced symmetry points it is useful to focus first on the long roots, which have $p_1 = 0$, $p_2 \in \sqrt{2}\Gamma^8$. We will now construct an orthonormal set of long roots, with inner product $e_{(i)} \cdot e_{(j)} = 2\delta_{ij}$, that extends the basis $(e_{(1)}, e_{(2)})$ described above to the root-lattice of $(SU(2))^{10}$. To begin with, we identify a useful basis of vectors contained in the euclidean lattice $\sqrt{2}\Gamma^8$. These are

$$\begin{aligned} u_{(i)} : \quad & p_2^1 = \sqrt{2}, \quad p_2^{i+1} = \sqrt{2}, \quad i = 1, \dots, 7 \\ u_{(8)} : \quad & p_2^I = \frac{1}{\sqrt{2}}, \quad I = 1, \dots, 8 \end{aligned} \quad (21)$$

with the property $u_{(i)} \cdot u_{(j)} = 2 + 2\delta_{ij}$. The vectors $e_{(1)}$, $e_{(2)}$, and

$$e_{(i)} : \quad p_2 = u_{(i-2)}, \quad p_3 = (0, \sqrt{2}, \sqrt{2}, \sqrt{2}), \quad i = 3, \dots, 10, \quad (22)$$

satisfy the orthonormality condition (19) and so by a Lorentz transformation can be taken to a left-moving basis, at which point they form the $(SU(2))^{10}$ root lattice normalized to level 1. The lattice $(SU(2))^n$ is the long root lattice of the non-simply laced group $Sp(2n)$. The linear combinations

$$\pm \frac{1}{2}e_{(i)} \pm' \frac{1}{2}e_{(j)} \quad (23)$$

are vectors of length 1 filling out the short root lattice of the group $Sp(20)_1$. These states can all be found in the untwisted sector, except for $i = 2$ or $j = 2$ which are contributed by the twisted sector.

Since we lack a more elegant characterization of the allowed momentum lattices, we will follow the above procedure in looking for points of enhanced symmetry. Fortunately, it suffices for the examples at hand. The possible enhanced symmetry points that can appear in the D=8 moduli space are limited. In the appendix we show that at any point in this moduli space the long roots are always orthogonal, so they can only form products of $SU(2)$'s. The only non-simply-laced groups that can appear are therefore $Sp(2n)$, including $Sp(4) = SO(5)$, while the only simply laced group that can appear at level 1 is a product of $SU(2)$'s.

Including an inner automorphism (Wilson line) in the translation leads to nothing new. That is, twist by RT' where now v_2 is nonzero. The requirement that T'^2 be a symmetry of Γ implies that $v_2 \in I^*$, but then the asymmetric orbifold generated by RT' has the same spectrum as that generated by RT .

It is interesting to consider the decompactification limit of the $Sp(20)$ theory in which the radius of the 9-direction is taken to infinity. Initially the gauge symmetry is broken to $Sp(18)$, but in the limit the twist becomes irrelevant and the antisymmetric combinations

$$\frac{1}{\sqrt{2}}(\alpha_{-1}^I - \alpha_{-1}^{II})|0, 0, 0\rangle, \quad \frac{1}{\sqrt{2}}\left(\left|\frac{1}{\sqrt{2}}r, \frac{1}{\sqrt{2}}r, 0\right\rangle - \left|-\frac{1}{\sqrt{2}}r, \frac{1}{\sqrt{2}}r, 0\right\rangle\right) \quad (24)$$

become massless. These lie in the antisymmetric tensor representation of $Sp(18)$, combining with the $Sp(18)$ adjoint to form the adjoint of $SU(18)$, a Narain compactification. Decompactifying the remaining direction along a line of unbroken $SU(16)$ leads in the limit to $SO(32)$, so these theories can also be regarded as compactifications of the $SO(32)$ string.

2.2 Compactification to $D < 8$

As D decreases, more gauge groups become possible. It is convenient to take $\Gamma' = \Gamma^{SU(2)} \oplus \Gamma^{9-D, 9-D}$. Here $\Gamma^{SU(2)}$ is the self-dual $SU(2)$ lattice described above, $(p_3^9; \tilde{p}_3^9)$. Let w, \tilde{w} denote vectors in the weight lattice of $SO(18-2D)$. Then $\Gamma^{9-D, 9-D}$ is defined as the lattice $(w; \tilde{w})$ such that $w - \tilde{w}$ is in the root lattice of $SO(18-2D)$. Momenta p_3 are thus labeled $(w, p_3^9; \tilde{w}, \tilde{p}_3^9)$. Define an orthonormal set $w_{(i)}$ of $SO(18-2D)$ vector weights,

$$w_{(i)} \cdot w_{(j)} = \delta_{ij}. \quad (25)$$

For T take the shift⁷

$$v_2 = 0, \quad v_3 = (w_{(1)}, 0; 0, 0). \quad (26)$$

Define the following:

$$\begin{aligned} f_{(i)} : \quad p_2 &= u_{(i)}, \quad p_3 = (0, 0; 0, \sqrt{2}), \quad i = 1, \dots, 8, \\ f_{(9)} : \quad p_2 &= 0, \quad p_3 = (0, \sqrt{2}; 0, 0), \\ g_{(i)} : \quad p_2 &= u_{(i)}, \quad p_3 = (0, 0; w_{(i)}, \sqrt{2}), \quad i = 1, \dots, 9 - D, \\ h_{(i)} : \quad p_2 &= 0, \quad p_3 = (w_{(i)}, 0; 0, 0), \quad i = 1, \dots, 9 - D. \end{aligned} \quad (27)$$

All of these are allowed momenta, g and h being in the twisted sector. For $1 \leq n \leq 10 - D$ the set

$$f_{(i)}, \quad i = n, \dots, 9, \quad g_{(i)}, \quad i = 1, \dots, n - 1, \quad h_{(i)}, \quad i = 1, \dots, 9 - D, \quad (28)$$

is a basis of $18 - D$ orthogonal vectors, with the f 's having length-squared 2 and the g 's and h 's having length-squared 1. By an $SO(18 - D, 10 - D)$ transformation, bring the given set to lie fully on the left. The f 's form the long roots of $Sp(20 - 2n)_1$ and the g 's and h 's form the short roots of $SO(17 - 2D + 2n)_1$. The remaining roots of each group (the sums and differences of the short roots, and $\frac{1}{2}$ the sums and differences of the long roots) are all in the untwisted sector. The product $Sp(20)_1 \times SO(17 - 2D)_1$, obtained first in the fermionic construction, is also in the same moduli space. Replacing $h_{(9-D)}$ with

$$f_{(10)} : \quad p_2 = \frac{1}{\sqrt{2}}(5, 1^7), \quad p_3 = (2w_{(9-D)}, 0; 0, 3\sqrt{2}) \quad (29)$$

gives the needed long root of $Sp(20)$, the additional short roots being in the twisted sector. Thus we have found

$$Sp(20 - 2n)_1 \times SO(17 - 2D + 2n)_1 \quad n = 0, \dots, 10 - D \quad (30)$$

at special points in a single moduli space.

The maximal symplectic group is $Sp(20)$. The central charge of $Sp(2n)_1$ is $(2n^2 + n)/(n + 2)$. The minimum central charge, including an additional $18 - n - D$ from $U(1)$'s needed to saturate the left-moving rank, exceeds the available $26 - D$ if $n > 10$. We also strongly believe that $SO(37 - 4D)$ is maximal. It might appear that one could get larger $SO(2n + 1)$ groups by decompactification, say $SO(25)$ in $D = 4$ from

⁷For $D = 8$ this is equivalent to the earlier construction under a boost and Weyl reflection.

$SO(25)$ in $D = 3$. However, this idea would lead to a contradiction in going from $D = 7$ to $D = 8$, where we have already shown that $SO(5)$ is maximal. In fact, study of these decompactifications shows that decompactifying while preserving the larger orthogonal symmetry gives a limit in which additional vectors become massless, leading to a rank 16 Narain theory. We expect that the $D = 8$ result can be extended to show that $SO(37 - 4D)$ is maximal.

While larger Sp and SO groups cannot arise, there are other possibilities. For example, twisting the $D=6$ $Sp(16)_1 \times SO(9)_1$ theory by the additional T' ,

$$v' = \frac{1}{2}(f_{(2)} + f_{(3)} + f_{(4)} + f_{(5)}) \quad (31)$$

leads to the gauge group $(F_4)_1 \times (F_4)_1 \times SO(9)_1$. This is in the same moduli space: we could twist by T' before RT , producing a different toroidal starting point which must therefore be related by a boost [1]. Similarly the group $(F_4)_1 \times (F_4)_1 \times Sp(8)_1$ can be obtained [6].

3 Discussion

Twisting by the outer isomorphism reduces the rank by eight relative to toroidal compactification, giving the moduli space of the rank -8 theory. This orbifold construction appears to reproduce all of the rank -8 solutions obtained in the fermionic construction [6],[7] and shows that they are indeed special points within a single moduli space. A more elegant characterization of the allowed lattices analogous to that of ref. [1] would be helpful.⁸

As mentioned in the introduction, this is only the first member in a series of maximally supersymmetric theories characterized by further reduction of the dimension of the moduli space, and it remains to find a construction of the moduli spaces of the remaining theories. Since Majorana fermion field theories only contain Z_2 spin fields, it is possible that an asymmetric orbifold construction of these moduli spaces will turn up additional reduced rank moduli spaces that are absent in the fermionic construction. Conversely, it is also possible that the solutions with rank $r_L < 10 - D$

⁸The global *reduction* of the rank (dimension) of a maximally supersymmetric string moduli space via orbifolding should be contrasted with familiar field theoretic mechanisms for symmetry enhancement at specific points in a moduli space, such as Higgsing, whether by fundamental scalars or more exotic composites.

in four dimensions have no orbifold realization. In these cases, the marginal deformations of the six dimensional abelian torus are either absent or partly constrained. These backgrounds do not appear to have a large radius limit.

It is quite interesting to consider the implications of the CHL string for weak/strong coupling duality. Dual theories describing the strongly coupled limit of the heterotic string in various dimensions have been proposed; see refs. [2], [3], [4] and references therein. In $D = 4$ the conjectured dual is the heterotic string itself [12], with the evidence being strongest in the case of toroidal compactification [2], [4]. What are the duals of the CHL theories? This presents a new challenge, because the S-duality transformation includes electric-magnetic duality of the low energy theory [13], and the groups $Sp(2n)$ and $SO(2n + 1)$ are not invariant under electric-magnetic duality but rather are interchanged [14]. Thus, S-duality cannot leave the individual points in the CHL moduli space invariant, but requires that for each solution there be a heterotic string solution with the dual group. The construction in the previous section seems in no way to single out $D = 4$, and it is evident from the list (30) that it does not automatically give dual groups. But remarkably, just in $D = 4$ where required by S-duality, the series (30) is dual: the strongly coupled behavior at one of these special points in moduli space can be described by the weakly coupled theory with $n \rightarrow 6 - n$, the point $n=3$ being self-dual. This is further evidence for S-duality, apparently independent of previous results. Moreover it is evidence for duality of the full string theories, not the low energy effective theories, because stringy phenomena (enhanced gauge symmetries) play an essential role. It would be of course interesting to compare the spectrum of short multiplets of these two theories.

In $D = 6$ there is a puzzle, however. There is mounting evidence for string-string duality, the heterotic string on T^4 being equivalent to the IIA string on K_3 [3], [4]. It is natural to look for a related dual in the present case. Indeed, Ferrara, Harvey, Strominger, and Vafa [10] have recently considered a similar situation, in which the IIA string is compactified on $K_3 \times T^2$ modded by a Z_2 isomorphism. They argue that the dual theory is the heterotic string on an asymmetric orbifold of T^6 , where the reflection includes an outer isomorphism of $E_8 \times E_8$. In that case the reflection also acts on the right-movers, breaking half the supersymmetry, but it suggests that the dual in our case as well should be some twisting of the K_3 compactification of the IIA string.⁹ The following argument of Seiberg [15] appears to exclude this

⁹We would like to thank Paul Aspinwall and Andy Strominger for independently suggesting this

attractive possibility. In order to obtain $N=2$, $D=6$ supersymmetry it is necessary to preserve the $(4,4)$ world-sheet supersymmetry of the K_3 theory [5]. This is also a good background for the IIB string [15], giving chiral $N=2$ $D=6$ supergravity. Spacetime anomaly cancellation then determines completely the massless spectrum, in particular the number of massless vectors and moduli in the IIB theory. For the IIA on the same background this implies a rank 24 gauge group, exactly as found in toroidal compactification of the heterotic string but inconsistent with theories of reduced rank. But there are other possibilities; for example, the dual could be a type II theory with both spacetime supersymmetries right-moving, corresponding to a right-moving torus times a generic $N=1$ left-moving theory. The full picture of duality of the CHL theories remains to be discovered.

Acknowledgments

We would like to thank Paul Aspinwall, Jeff Harvey, Costas Kounnas, Joe Lykken, and Andy Strominger for illuminating discussions. S.C. acknowledges the warm hospitality of the CERN Theory division during the completion of this work. This work is supported by NSF grants PHY91-16964 and PHY94-07194.

Appendix

For the lattice used in $D = 8$ we show that if

$$p_1 = p'_1 = 0 \tag{A.1}$$

$$p \cdot p = 2 \bmod 4, \quad p' \cdot p' = 2 \bmod 4, \tag{A.2}$$

then

$$p \cdot p' = 0 \bmod 2. \tag{A.3}$$

Eq. (A.1) implies that p_2 and p'_2 lie on $\sqrt{2}\Gamma^8$ and so contribute to the lengths (A.2) only modulo 4 and to the dot product (A.3) only modulo 2. Similarly the RT projection implies that $n_9 + \tilde{n}_9$ is a multiple of 4, hence p^9 contributes in the same way. Finally, $n_8 = 2r - \tilde{n}_8$ for integer r . Then $p \cdot p \bmod 4$ is $\frac{1}{2}(n_8^2 - \tilde{n}_8^2) = 2r(r - \tilde{n}_8)$. It follows from (A.2) that r is odd and n_8 and \tilde{n}_8 even. Similarly n'_8 and \tilde{n}'_8 are even, and so eq. (A.3) holds.

and pointing out the parallel with ref. [10].

For long roots $p_1 = 0$ and $p \cdot p = 2$ with p purely left-moving and so having a positive inner product. It follows from this and eq. (A.3) that distinct long roots are orthogonal.

References

- [1] K. S. Narain, *Phys. Lett.* **169** (1986) 41;
K. S. Narain, M. Sarmadi and E. Witten, *Nucl. Phys.* **279B** (1987) 369.
- [2] A. Sen, *Int. Jour. Mod. Phys.* **A9** (1994) 3707, hep-th/9402002.
- [3] C. M. Hull and P. Townsend, *Unity of Superstring Dualities*, preprint QMW-94-30, R/94/33, hep-th/9410167;
M. J. Duff, *Strong/Weak Coupling Duality from the Dual String*, preprint NI-94-033, CTP-TAMU-49/94, hep-th/9501030;
J. Harvey and A. Strominger, *The Heterotic String is a Soliton*, preprint EFI-95-16, hep-th/9504047.
- [4] E. Witten, *String Theory Dynamics in Various Dimensions*, preprint IASSNS-HEP-95-18, hep-th/9503124.
- [5] T. Banks and L. Dixon, *Nucl. Phys.* **B307** (1988) 93.
- [6] S. Chaudhuri, G. Hockney and J. Lykken, *Maximally Supersymmetric String Theories in $D < 10$* , preprint FERMI-PUB-95/099-T, NSF-ITP-95-37, hep-th/9505054.
- [7] S. Chaudhuri, Talk given at Strings '95, USC, March 95, proceedings to appear.
- [8] S. Ferrara and C. Kounnas, *Nucl. Phys.* **B328** (1989) 406.
- [9] L. Dixon and J. Harvey, *Nucl. Phys.* **274** (1986);
H. Kawai, D. Lewellen, and S.-H. H. Tye, *Phys. Rev.* **D34** (1986) 3794;
S. Elitzur and A. Giveon, *Phys. Lett.* **189** (1987) 52;
P. Forgacs, Z. Horvath, L. Palla, and P. Vecsernyes, *Nucl. Phys.* **B308** (1988) 477;
A. Font, L. Ibanez, F. Quevedo, and A. Sierra, *Nucl. Phys.* **331** (1990) 421.

- [10] S. Ferrara, J. Harvey, A. Strominger, and C. Vafa, *Second Quantized Mirror Symmetry*, preprint EFI-95-26, hep-th/9505162.
- [11] K. S. Narain, M. H. Sarmadi, and C. Vafa, *Nucl. Phys.* **288** (1987) 551.
- [12] A. Font, L. Ibanez, D. Lust, and F. Quevedo, *Nucl. Phys.* **B249** (1990) 35.
- [13] C. Montonen and D. Olive, *Phys. Lett.* **72B** (1977) 117;
 E. Witten and D. Olive, *Phys. Lett.* **78B** (1978) 97;
 H. Osborne, *Phys. Lett.* **83B** (1979) 321.
- [14] P. Goddard, J. Nuyts, and D. Olive, *Nucl. Phys.* **B125** (1977) 1.
- [15] N. Seiberg, *Nucl. Phys.* **B303** (1988) 286.